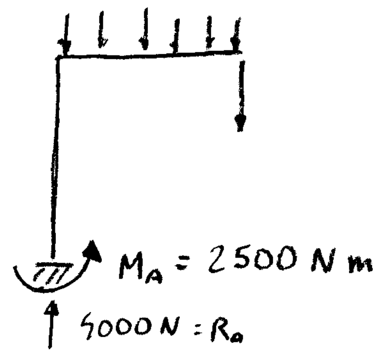
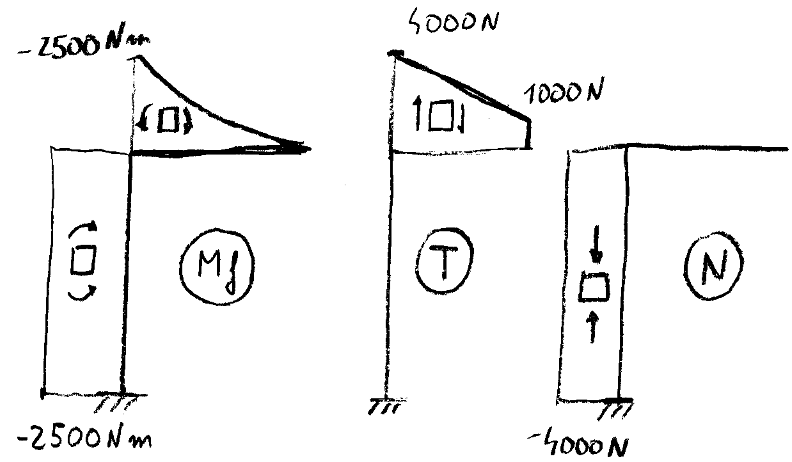
ELASTICIDAD Y RESISTENCIA DE MATERIALES. 2º SEMESTRE. PLAN 1976
26-6-02

1.

REACCIONES:





2.

$$\Delta l = \frac{\sigma l}{E} = \frac{N l}{E \Omega} = \frac{N_1 l}{E_1 \Omega_1} = \frac{N_2 l}{E_2 \Omega_2} \Rightarrow N_1 = N_2 \frac{E_1 \Omega_1}{E_2 \Omega_2}$$

Además, por la condición de equilibrio: $N_1 + N_2 = 30$ kN

$$N_1 = (30\text{kN} - N_1)\frac{E_1 \Omega_1}{E_2 \Omega_2} \Rightarrow N_1 = \frac{30\text{kN} \cdot E_1 \Omega_1}{E_1 \Omega_1 + E_2 \Omega_2} = \frac{30 \cdot 40 \cdot 50400}{40 \cdot 50400 + 205 \cdot 9600} = 15\text{kN}$$

$\Omega_1 = 50400$ mm²

$\Omega_2 = 9600$ mm²

$$\Rightarrow \boxed{\Delta l} = \frac{N_1 l_1}{E_1 \Omega_1} = \frac{15 \cdot 2000}{40 \cdot 50400} = \boxed{0{,}015 \text{ mm}}$$

3.

En cada tramo en el que el momento torsor es cte., tenemos:

TRAMO 1-2: $\theta_{1-2} = \dfrac{M_{12} \cdot l_{12}}{G\, I_{012}} \Rightarrow M_{12} = \dfrac{\theta_{12}\, G\, I_{012}}{l_{12}} = \dfrac{-0{,}002 \cdot 8 \cdot 10^4 \cdot 981{,}7}{100}$

$I_{12} = \dfrac{\pi d_{12}^4}{32} = 981{,}7$ mm⁴ $\Rightarrow M_{12} = 1{,}57$ Nm

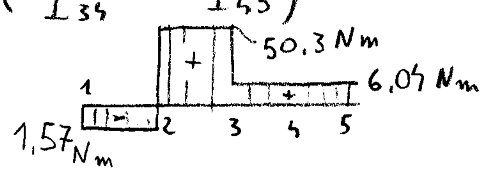
$I_{23} = 15707$ mm⁴ TRAMO 2-3: $M_{23} = \dfrac{\theta_{23}\, G\, I_{23}}{l_{23}} = \dfrac{0{,}008 \cdot 8 \cdot 10^4 \cdot 15707}{200} \Rightarrow$

$\Rightarrow M_{23} = 50{,}3$ Nm

$I_{34} = I_{23}$

$I_{45} = 4970$ mm⁴ TRAMO 3-5: $\theta_{35} = \dfrac{M_{35}}{G}\left(\dfrac{l_{34}}{I_{34}} + \dfrac{l_{45}}{I_{45}}\right) \Rightarrow$

$$M_{35} = \frac{\theta_{35} \cdot G}{\frac{l_{34}}{I_{34}} + \frac{l_{45}}{I_{45}}} = \frac{0{,}002 \cdot 8 \cdot 10^4}{\frac{100}{15707} + \frac{100}{4970}} = 6{,}04 \text{Nm}$$



4. La fórmula de Euler es aplicable en el dominio elástico, por lo que la esbeltez no puede ser inferior a: $\lambda \geq \pi\sqrt{\frac{E}{\sigma_e}}$. Además $\lambda = \frac{l_p}{\sqrt{I_z/\Omega}}$

a)

$$\Rightarrow \frac{l_p}{\sqrt{I_z/\Omega}} \geq \pi\sqrt{\frac{E}{\sigma_e}} \Rightarrow l_p \geq \pi\sqrt{\frac{E\,I_z}{\sigma_e \cdot \Omega}}$$

Como

$$\left.\begin{array}{l} I_z = \frac{\pi(R_e^4 - R_i^4)}{4} \\ \Omega = \pi(R_e^2 - R_i^2) \\ l = 2 l_p \end{array}\right\} l \geq 2\pi\sqrt{\frac{E\cdot(R_e^4 - R_i^4)}{\sigma_e \cdot 4(R_e^2 - R_i^2)}} = \boxed{4{,}88\ m}$$

b)

$\varepsilon_x = \alpha\Delta T = \frac{\sigma_{nx}}{E} \Rightarrow \sigma_{nx} = E\alpha\Delta T$. Será inestable si $\sigma_{nx} \geq \sigma_{cr}$ y en este caso (para $l = 4{,}88\ m$) $\sigma_{cr} = \sigma_e$. Por tanto

$$E\alpha\Delta T = 205 \cdot 11{,}7\cdot10^{-6}\cdot 50 = 0{,}12\ GPa < 0{,}24\ GPa \Rightarrow \underline{\text{ES ESTABLE}}$$

5. De la condición a) $\rightarrow$ $\tau = \frac{M_T}{I_o} R \leq \tau_{adm} \Rightarrow \boxed{\tau_{adm}} \geq \frac{2\cdot10^6}{\frac{\pi\cdot40^4}{2}} = \boxed{19{,}9\ MPa}$

de b) $\rightarrow$ $\theta = \int_0^1 \frac{M_T\,dx}{G\,I_o} = \int_0^1 \frac{2000\cdot(1-x)dx}{G\cdot\pi\cdot\frac{40^4}{2}} = 0{,}5\cdot\frac{\pi}{180} \Rightarrow$

$$\Rightarrow G = \frac{2\cdot2\cdot10^6\cdot180}{\cancel{2}\cdot\pi^2\cdot40^4\cdot\cancel{0{,}5}} = \boxed{28{,}5\ GPa}$$