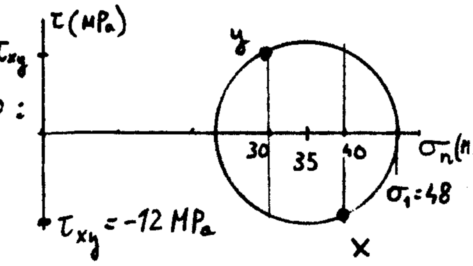
ELASTICIDAD Y RESISTENCIA (PLAN 76). 1^ER SEMESTRE. 13-2-2004

CUESTIONES

1. Dado que $\left.\begin{array}{l}\sigma_{nx} = 40 \text{ MPa} \\ \sigma_{ny} = 30 \text{ MPa}\end{array}\right\}$ El centro del círculo de Mohr se sitúa en $\sigma = 35$ MPa

Como además $\sigma_1 = 48$ MPa, deberá pasar por ese punto:

Del círculo se obtiene igualmente $\tau_{xy} = 12$ MPa



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2. Se necesita la roseta tipo A, pues son 3 las incógnitas (ε_x, ε_y y γ_{xy}) y cada galga proporciona una ecuación

$$\varepsilon_n = \varepsilon_x \alpha^2 + \varepsilon_y \beta^2 + \gamma_{xy}\alpha\beta \left\{\begin{array}{lll} a & \left\{\begin{array}{l}\alpha = 0\\ \beta = 1\end{array}\right\} & \varepsilon_a = \varepsilon_y \\ b & \left\{\begin{array}{l}\alpha = 1/\sqrt{2}\\ \beta = 1/\sqrt{2}\end{array}\right\} & \varepsilon_b = \varepsilon_x/2 + \varepsilon_y/2 + \gamma_{xy}/2 \\ c & \left\{\begin{array}{l}\alpha = 1\\ \beta = 0\end{array}\right\} & \varepsilon_c = \varepsilon_x \end{array}\right.$$

$$\Rightarrow \boxed{\gamma_{xy}} = 2\varepsilon_b - \varepsilon_a - \varepsilon_c = \boxed{32\cdot 10^{-6} \text{ rad}}$$

Leyes de Hooke: $\left[\varepsilon_x = \frac{1}{E}(\sigma_{nx} - \mu\sigma_y)\right] * \mu$ $\quad \gamma_{xy} = \frac{\tau_{xy}}{G} \Rightarrow \boxed{\tau_{xy}} = \gamma_{xy}\cdot G = \boxed{2{,}46 \text{ MPa}}$

$$\varepsilon_y = \frac{1}{E}(\sigma_{ny} - \mu\sigma_{nx}) \qquad G = \frac{E}{2(1+\mu)} = 0{,}769\cdot 10^5 \text{ MPa}$$

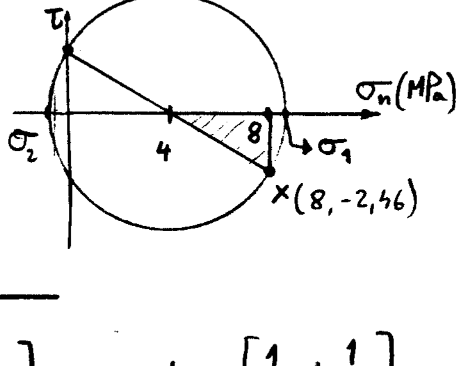
$$\mu\varepsilon_x + \varepsilon_y = \frac{1}{E}(-\mu^2\sigma_{ny} + \sigma_{ny}) \Rightarrow \boxed{\sigma_{ny}} = \frac{E(\mu\varepsilon_x + \varepsilon_y)}{1-\mu^2} = \boxed{0} \Rightarrow$$

$$\Rightarrow \boxed{\sigma_{nx} = 8 \text{ MPa}}$$

Las tensiones principales se obtienen del círculo de Mohr:

$$\sigma_1 = \sqrt{4^2 + 2{,}46^2} + 4 = 8{,}697 \text{ MPa}$$

$$\sigma_2 = 4 - \sqrt{4^2 + 2{,}46^2} = -0{,}697 \text{ MPa}$$



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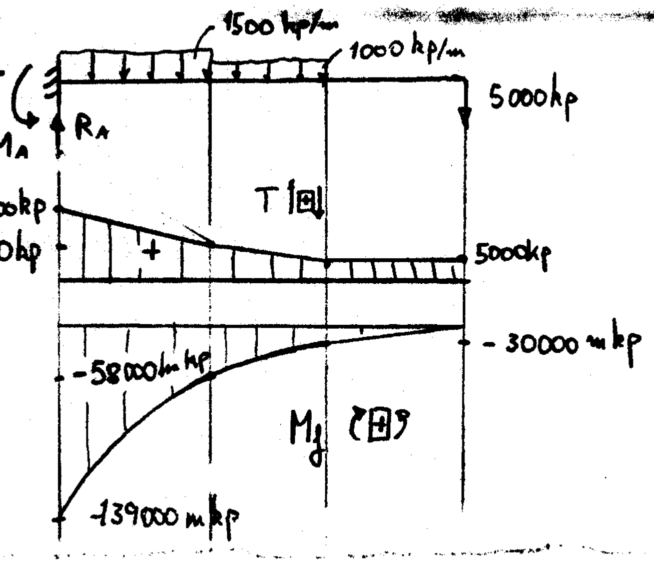
3. Caso (A): $$\mathcal{E}_A = \sum_{i=1}^{2} \frac{P^2\cdot l_i}{2E\Omega_i} = \frac{P^2 h \cdot 4}{2E\pi\, 2}\left[\frac{1}{D^2} + \frac{1}{d^2}\right] = K\cdot\left[\frac{1}{D^2} + \frac{1}{d^2}\right]$$

Caso (B) $$\mathcal{E}_B = \int_0^h \frac{P^2\,dx}{2E\cdot\Omega(x)} = \frac{P^2\cdot 4}{2E\pi}\int \frac{dx}{\left[d + \left(\frac{D-d}{h}\right)x\right]^2}$$ Tomando como variable de integración $u = d + \left(\frac{D-d}{h}\right)x$ resulta:

$$\mathcal{E}_B = \frac{P^2\cdot h\cdot 2}{E\pi(D-d)}\int\frac{du}{u^2} = K\cdot\frac{2}{(D-d)}\left[\frac{-1}{u}\right] = K\cdot\frac{2}{(d-D)}\left[\frac{1}{d + \left(\frac{D-d}{h}\right)x}\right]_0^h = K\cdot\frac{2}{D\cdot d}$$

Por tanto $$\frac{\mathcal{E}_A}{\mathcal{E}_B} = \frac{1}{2}\left[\frac{d}{D} + \frac{D}{d}\right] \Rightarrow \left\{\begin{array}{l}\text{Si } D = d \Rightarrow \mathcal{E}_A = \mathcal{E}_B \text{ (Trivial)} \\ \text{Si } D > d \text{ (caso general)} \Rightarrow \boxed{\mathcal{E}_A > \mathcal{E}_B}\end{array}\right.$$

4.



$\underline{R_A = 18000 kp}$

$M_A = 1500 \cdot 6 \cdot 3 + 1000 \cdot 4 \cdot 8 + 5000 \cdot 16 =$

$\underline{M_A = 139000 \text{ m kp}}$

5 a) No es necesaria ninguna otra acción por existir ya equilibrio

b) Ensayamos una solución del tipo $\phi = Ax^3 + Bx^2y + Cxy^2 + Dy^3 + Ex^2 + Fxy + Gy^2$

$$\left.\begin{aligned} \sigma_{nx} &= \frac{\partial^2 \phi}{\partial y^2} = 2Cx + 6Dy + 2G \\ \sigma_{ny} &= \frac{\partial^2 \phi}{\partial x^2} = 6Ax + 2By + 2E \\ \tau_{xy} &= -2Bx - 2Cy + F \end{aligned}\right\}$$

CARA AD $\quad \sigma_{ny} = -30x$

$(y=0) \rightarrow -30x = 6Ax + 2E \Rightarrow A = -5, \; E = 0$

$\tau_{xy} = 0 \Rightarrow B = 0, \; F = 0$

CARA AB $\quad \sigma_{nx} = -36y$

$(x=0) \rightarrow -36y = 6Dy + 2G \Rightarrow D = -6, \; G = 0$

$\tau_{xy} = 0 = -2Cy + F \Rightarrow \begin{cases} C = 0 \\ F = 0 \end{cases}$

$\rightarrow \underline{\underline{\phi = -5x^3 - 6y^2}}$ (x, y en cm, Tensiones en MPa)