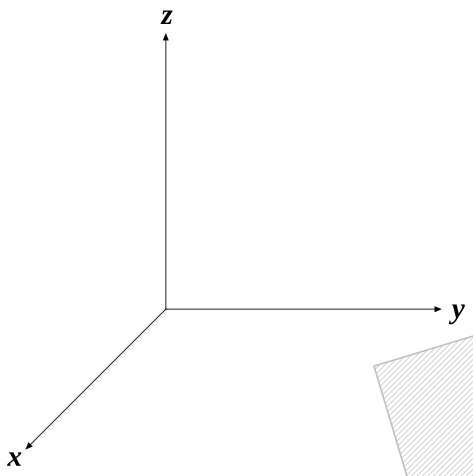
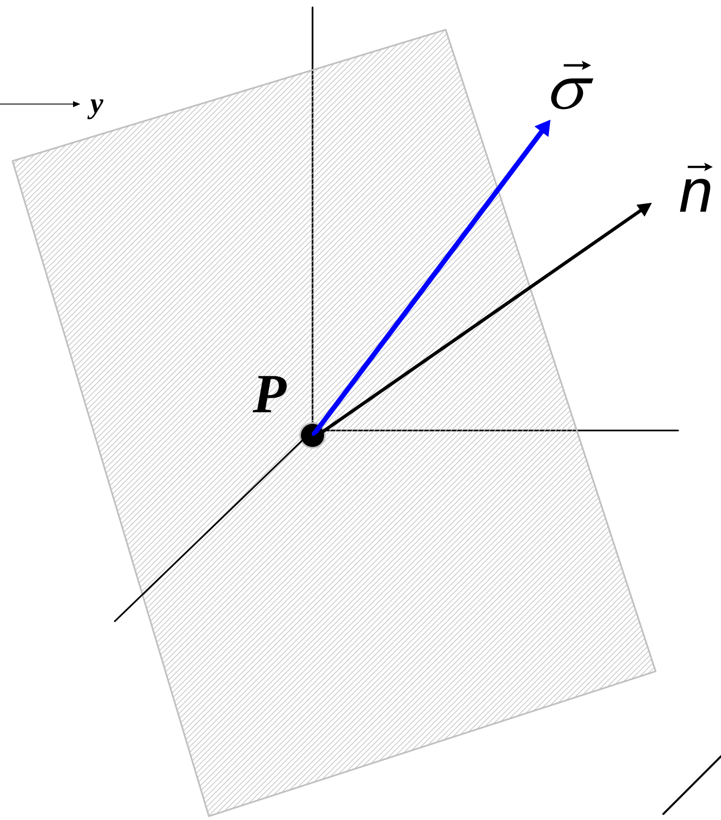
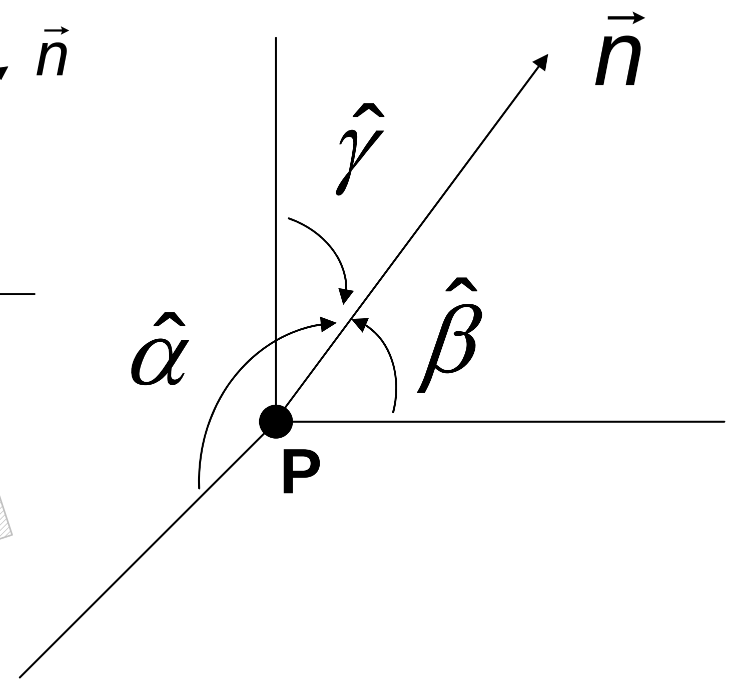
VECTOR TENSIÓN PARA UNA ORIENTACIÓN CUALQUIERA: MATRIZ DE TENSIONES

$$\vec{n} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} \qquad \begin{aligned} \alpha &= \cos\hat{\alpha} \\ \beta &= \cos\hat{\beta} \\ \gamma &= \cos\hat{\gamma} \end{aligned}$$

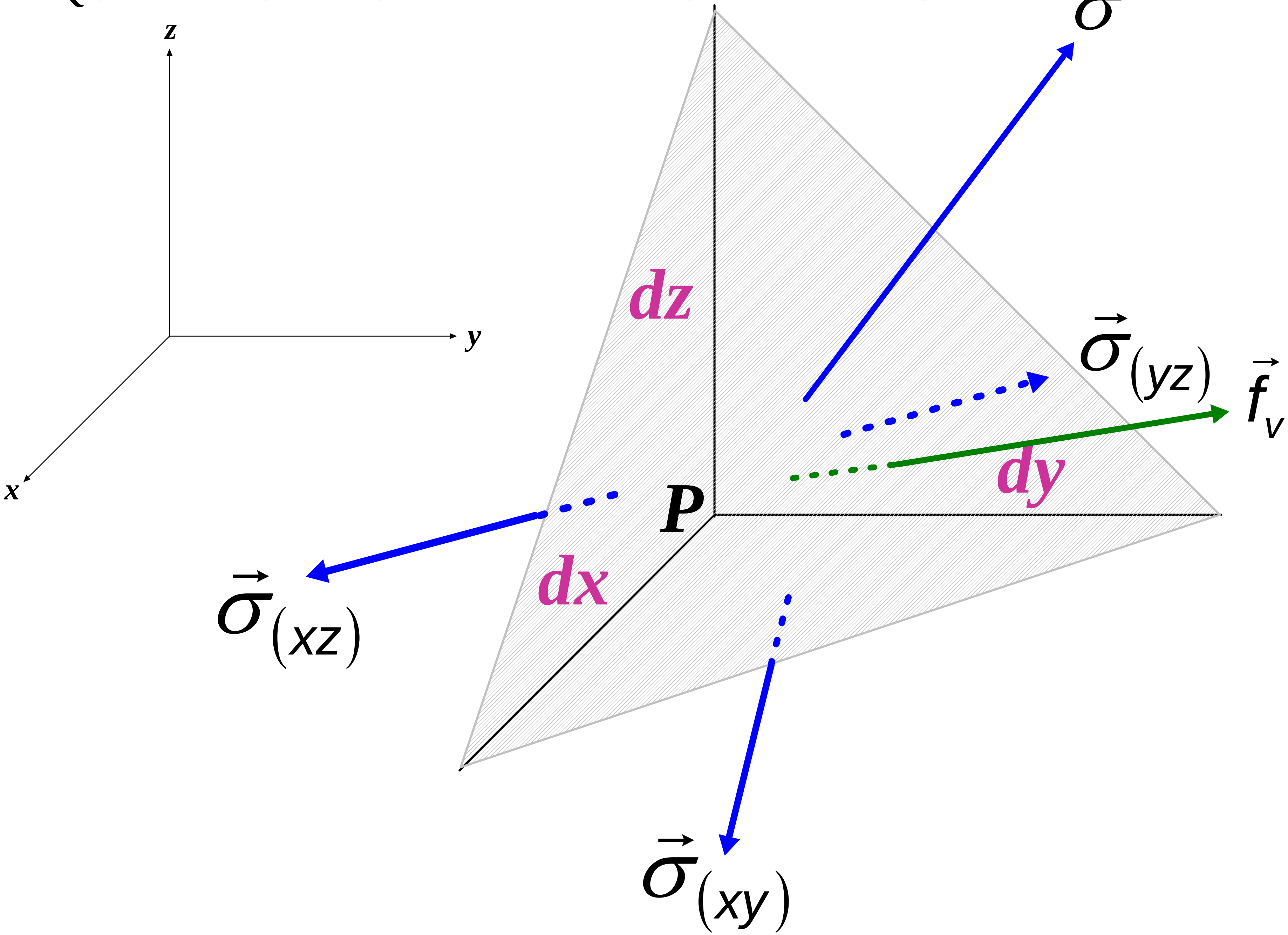






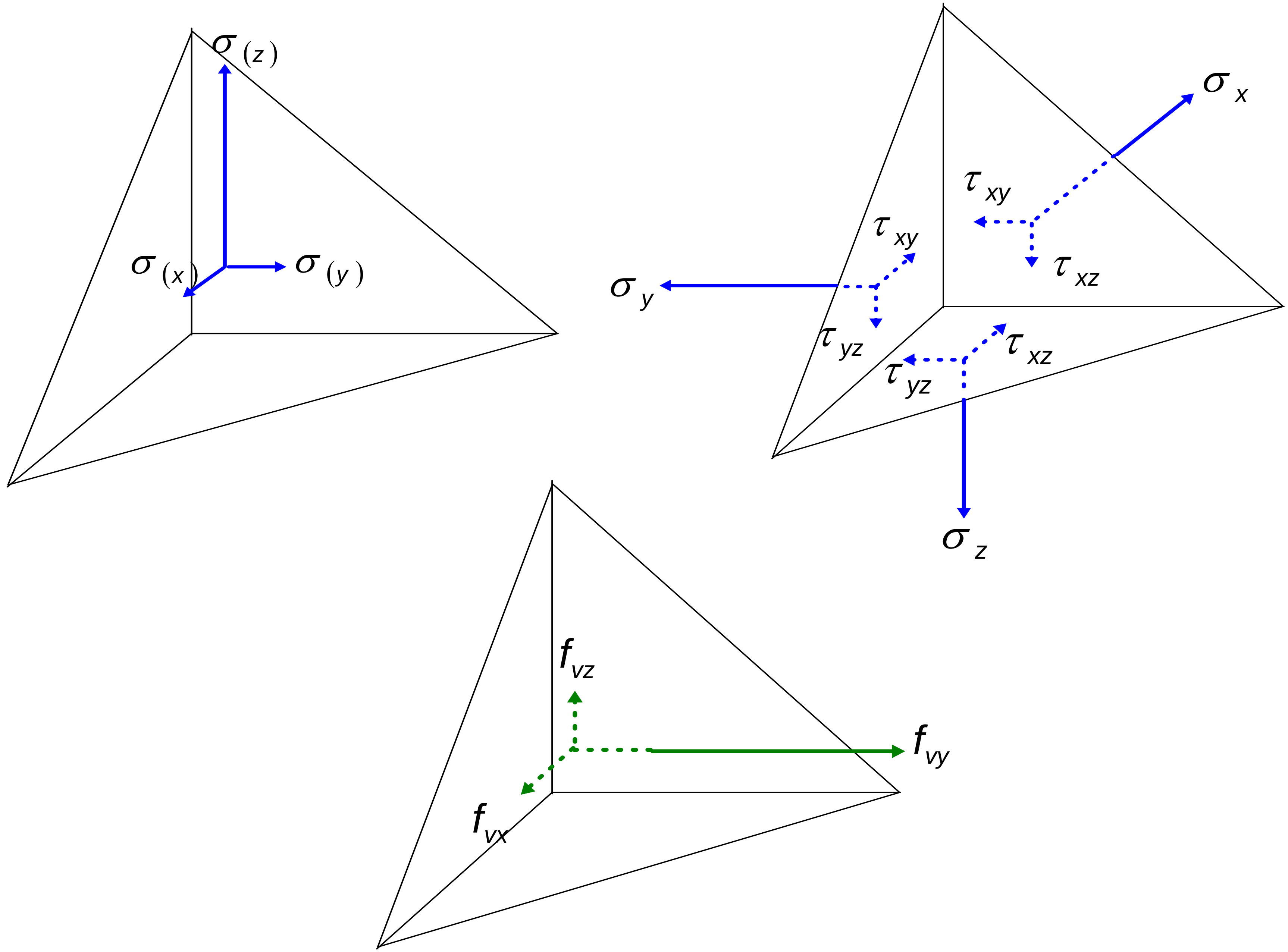
Es posible conocerlo si se conocen los vectores tensión correspondientes a los planos coordenados

EQUILIBRIO EN UN TETRAEDRO DIFERENCIAL

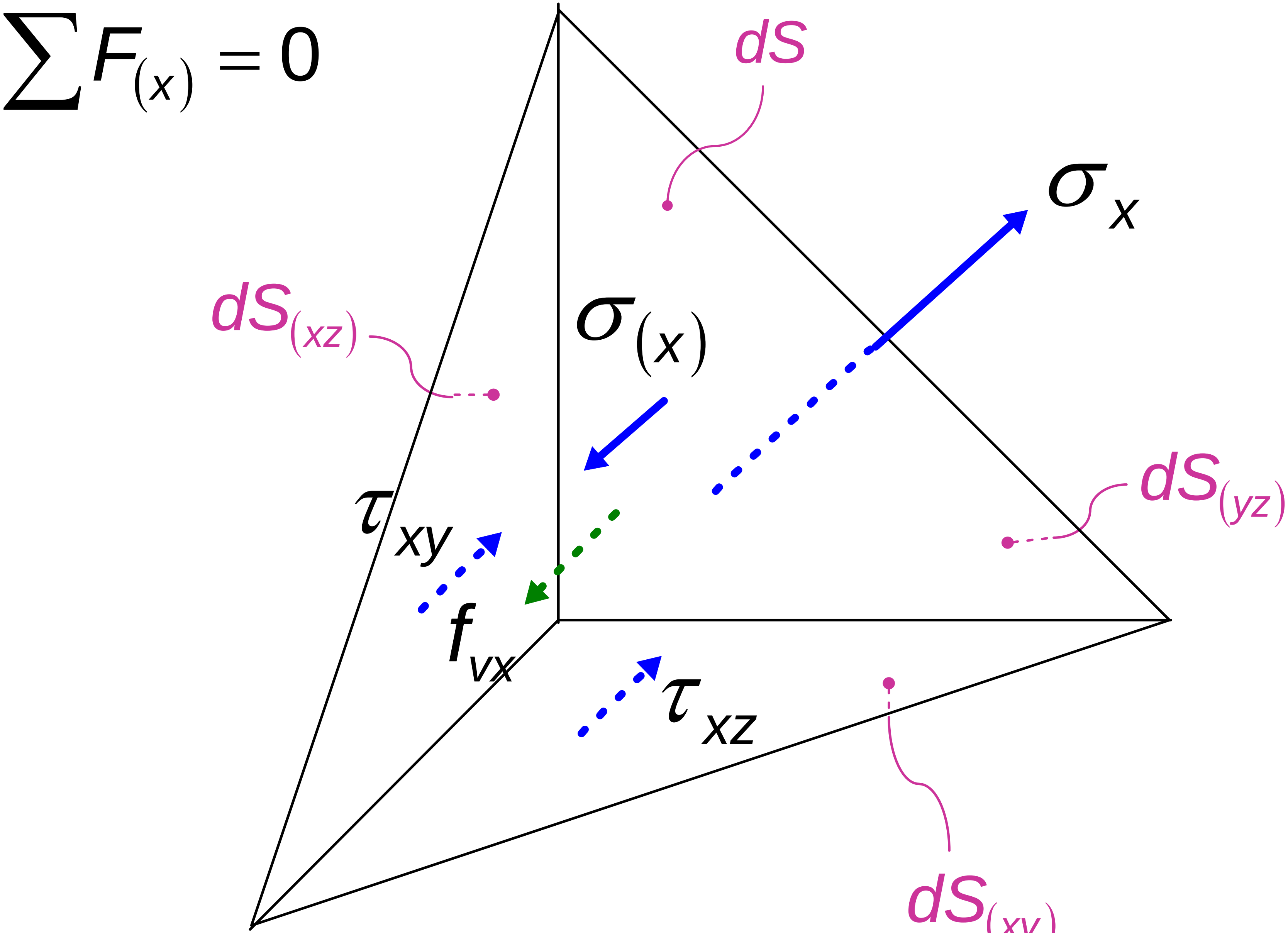


EQUILIBRIO DE FUERZAS

$$\sum \vec{F} = \vec{0} \quad \longrightarrow \quad \begin{cases} \sum F_{(x)} = 0 \\ \sum F_{(y)} = 0 \\ \sum F_{(z)} = 0 \end{cases}$$

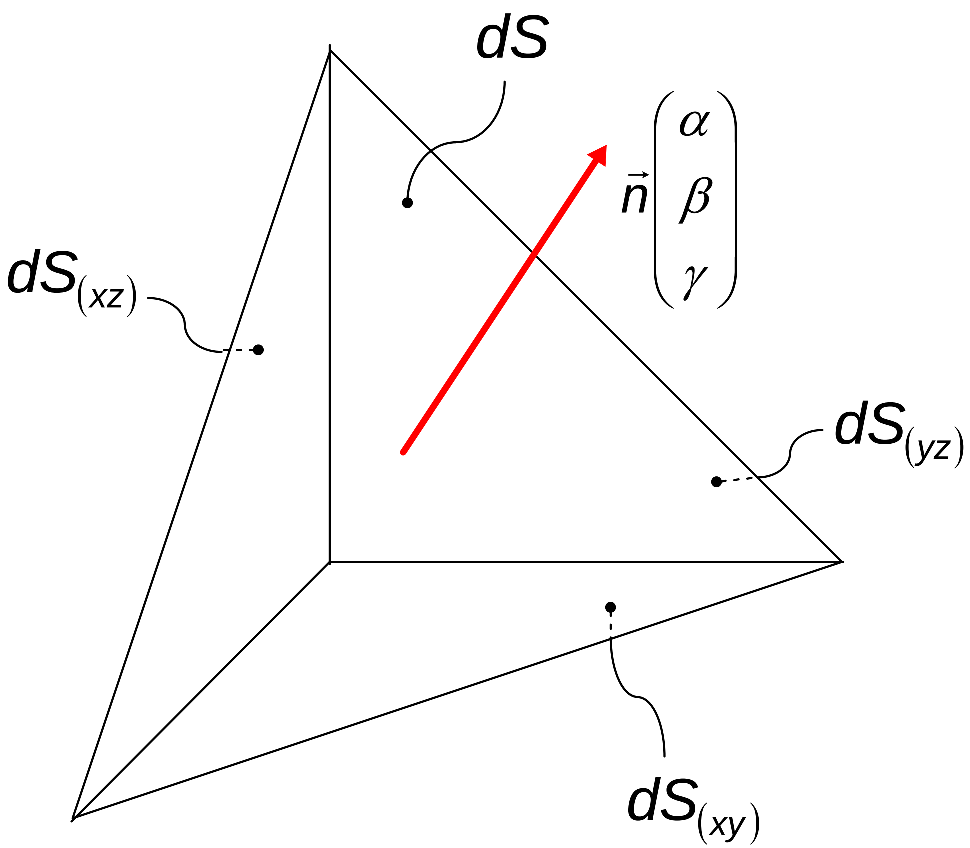


σ (z)
σ (x)
σ (y)
σ x
τ xy
τ xy
τ xz
σ y
τ yz
τ xz
τ yz
σ z
f vz
f vy
f vx


$\sum F_{(x)} = 0$
dS
σ_x
$dS_{(xz)}$
$\sigma_{(x)}$
$dS_{(yz)}$
τ_{xy}
f_{vx}
τ_{xz}
$dS_{(xy)}$

$$\sum F_x = 0$$

$$\sigma_{(x)} dS + f_{vx} dV - \sigma_x dS_{(yz)} - \tau_{yx} dS_{(xz)} - \tau_{zx} dS_{(xy)} = 0$$



$f_{vx} dV$: Infinitésimo de tercer orden (despreciable)

$$dS_{(yz)} = d\vec{S}\cdot\vec{i} =$$

$$= dS\cdot\left|\vec{i}\right|\cdot\alpha = dS\cdot\alpha$$

$$dS_{(xz)} = dS\cdot\beta$$

$$dS_{(xy)} = dS\cdot\gamma$$

Sustituyendo: $\sum F_x = 0 \rightarrow \quad \sigma_{(x)} = \sigma_x \alpha + \tau_{yx} \beta + \tau_{zx} \gamma$

Hay otras dos ecuaciones de equilibrio:

$$\sum F_Y = 0 \rightarrow \quad \sigma_{(y)} = \tau_{xy} \alpha + \sigma_y \beta + \tau_{zy} \gamma$$

$$\sum F_Z = 0 \rightarrow \quad \sigma_{(z)} = \tau_{xz} \alpha + \tau_{yz} \beta + \sigma_z \gamma$$

En formato matricial:

$$\begin{pmatrix} \sigma_{(x)} \\ \sigma_{(y)} \\ \sigma_{(z)} \end{pmatrix} = \begin{pmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$$

Abreviado:

$$\boxed{\vec{\sigma} = [T] \cdot \vec{n}}$$

[T] MATRIZ DE TENSIONES