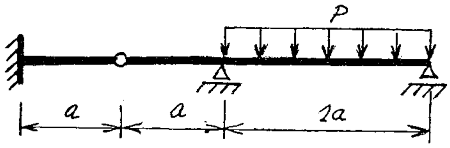
ESCUELA TÉCNICA SUPERIOR
DE
INGENIEROS INDUSTRIALES

UNIVERSIDAD POLITÉCNICA
DE
MADRID

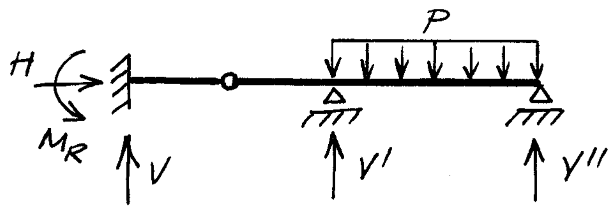
RESISTENCIA DE MATERIALES II. Curso 2006-07. **15/06/07**

EXAMEN DE JUNIO. PROBLEMA

Para la viga de sección constante indicada en la figura, se pide determinar las leyes de esfuerzos cortantes y momentos flectores, y dibujar los correspondientes diagramas acotados. *(10 puntos)*



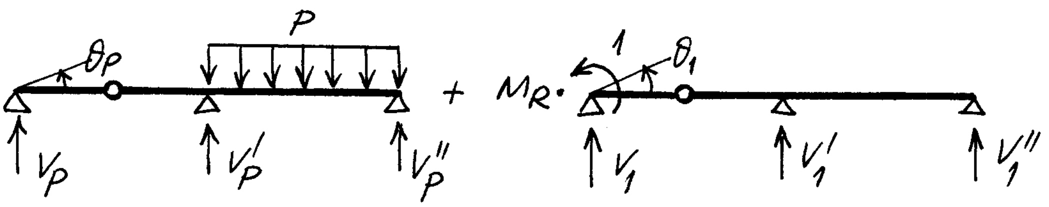
Las reacciones en los apoyos son $r = 5$; y las libertades $l = 1$



$(H = 0)$

$$GH = (r-3) + 3c - l = (5-3) + 0 - 1 = 1$$

Tomaremos como incógnita hiperestática el momento de reacción en el empotramiento M_R. Para determinarlo impondremos la condición de que el giro de la sección de empotramiento es nulo, aplicando superposición a los dos casos siguientes:



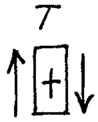
$$0 = \theta_P + M_R \cdot \theta_1$$

$$\left(\begin{array}{l} V_P + V'_P + V''_P - 2pa = 0 \\ V'_P \cdot 2a - 2pa \cdot 3a + V''_P \cdot 4a = 0 \\ V_P \cdot a = 0 \end{array} \right. \qquad \left(\begin{array}{l} V_1 + V'_1 + V''_1 = 0 \\ 1 + V'_1 \cdot 2a + V''_1 \cdot 4a = 0 \\ -1 + V_1 \cdot a = 0 \end{array} \right.$$

$$V_P = 0 \qquad V_1 = \frac{1}{a}$$

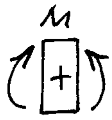
$$V'_P = V''_P = pa \qquad V'_1 = -\frac{3}{2a} \qquad V''_1 = \frac{1}{2a}$$

$$T_p = V_p + V_p'\langle x-2a\rangle^0 - p\langle x-2a\rangle = pa\langle x-2a\rangle^0 - p\langle x-2a\rangle$$

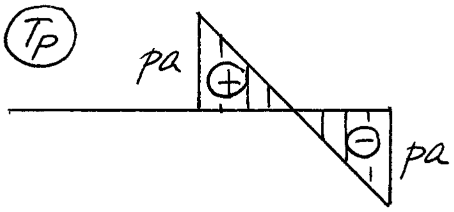


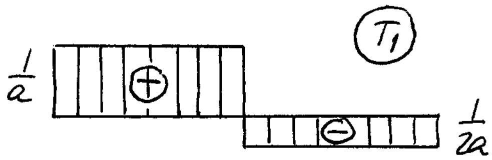
$$T_1 = V_1 + V_1'\langle x-2a\rangle^0 = \frac{1}{a} - \frac{3}{2a}\langle x-2a\rangle^0$$

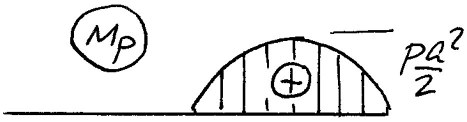
$$M_p = V_p\cdot x + V_p'\langle x-2a\rangle - \frac{p}{2}\langle x-2a\rangle^2 = pa\langle x-2a\rangle - \frac{p}{2}\langle x-2a\rangle^2$$

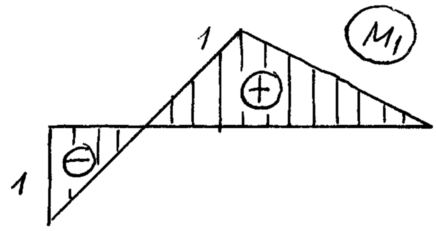


$$M_1 = -1 + V_1 x + V_1''\langle x-2a\rangle = -1 + \frac{1}{a}x - \frac{3}{2a}\langle x-2a\rangle$$









Para calcular θ_p y θ_1 aplicamos el método de la carga unitaria

$$\theta_p = \int \frac{M_p M_1}{EI}dx = \frac{1}{EI}\left(2a\frac{pa^2}{2} - 2\frac{1}{3}a\frac{pa^2}{2}\right)\cdot\frac{1}{2} = \frac{1}{EI}\frac{pa^3}{3}$$

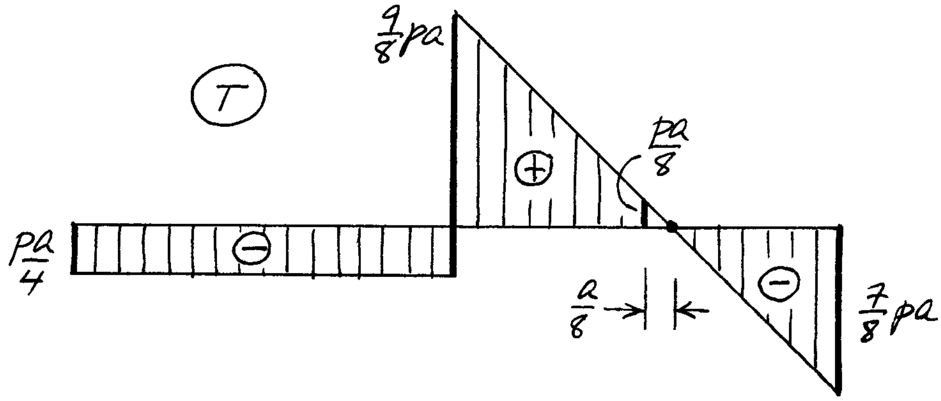
$$\theta_1 = \int \frac{M_1^2}{EI}dx = \frac{1}{EI}\left(\frac{1}{2}a\frac{2}{3} + \frac{1}{2}a\frac{2}{3} + \frac{1}{2}2a\frac{2}{3}\right) = \frac{1}{EI}\frac{4a}{3}$$

Imponiendo la condición de giro nulo : $\theta_p + M_R\cdot\theta_1 = 0$

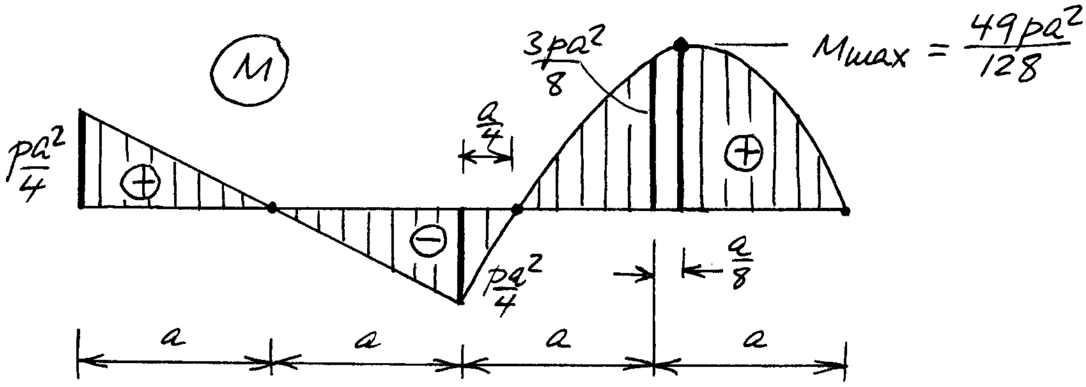
$$\frac{1}{EI}\cdot\frac{pa^3}{3} + M_R\cdot\frac{1}{EI}\frac{4a}{3} = 0 \longrightarrow M_R = -\frac{pa^2}{4}$$

$$T = T_p + M_R T_1 = pa\langle x-2a\rangle^0 - p\langle x-2a\rangle - \frac{pa}{4} + \frac{3pa}{8}\langle x-2a\rangle^0 = -\frac{pa}{4} + \frac{11pa}{8}\langle x-2a\rangle^0 - p\langle x-2a\rangle$$

$$M = M_p + M_R M_1 = pa\langle x-2a\rangle - \frac{p}{2}\langle x-2a\rangle^2 + \frac{pa^2}{4} - \frac{pa}{4}x + \frac{3pa}{8}\langle x-2a\rangle = \frac{pa^2}{4} - \frac{pa}{4}x + \frac{11pa}{8}\langle x-2a\rangle - \frac{p}{2}\langle x-2a\rangle^2$$



$$(2a,4a) \quad T = 0 = -\frac{pa}{4} + \frac{11pa}{8} - p(x-2a) \rightarrow x = 2a + \frac{9}{8}a =$$
$$= 3a + \frac{a}{8}$$



$$(2a,4a) \quad M_{max} = M\left(3a + \frac{a}{8}\right) = \frac{49}{128}pa^2$$

$$M = 0 = \frac{pa^2}{4} - \frac{pa}{4}x + \frac{11pa}{8}(x-2a) - \frac{p}{2}(x-2a)^2 \rightarrow x = 2a + \frac{a}{4}$$