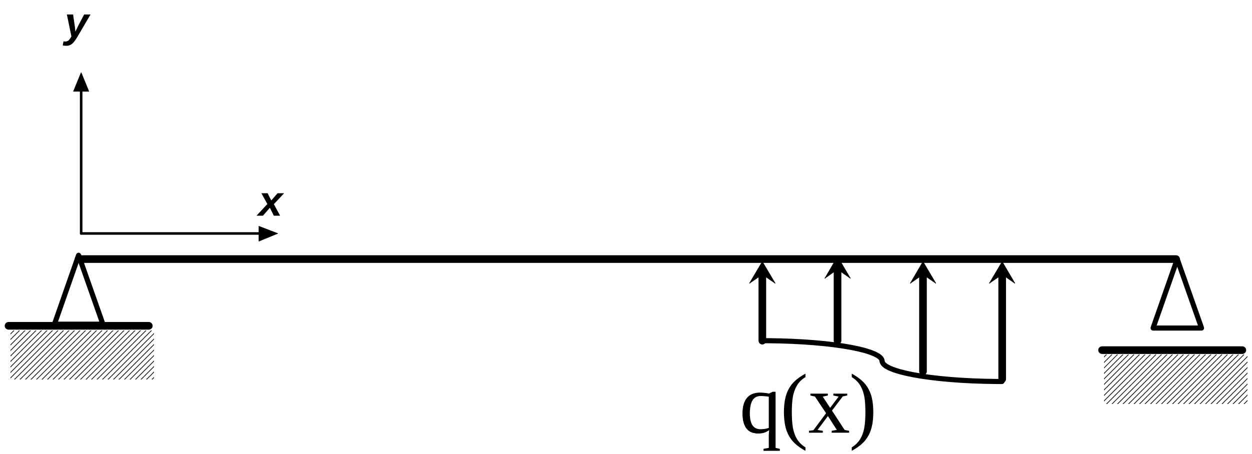
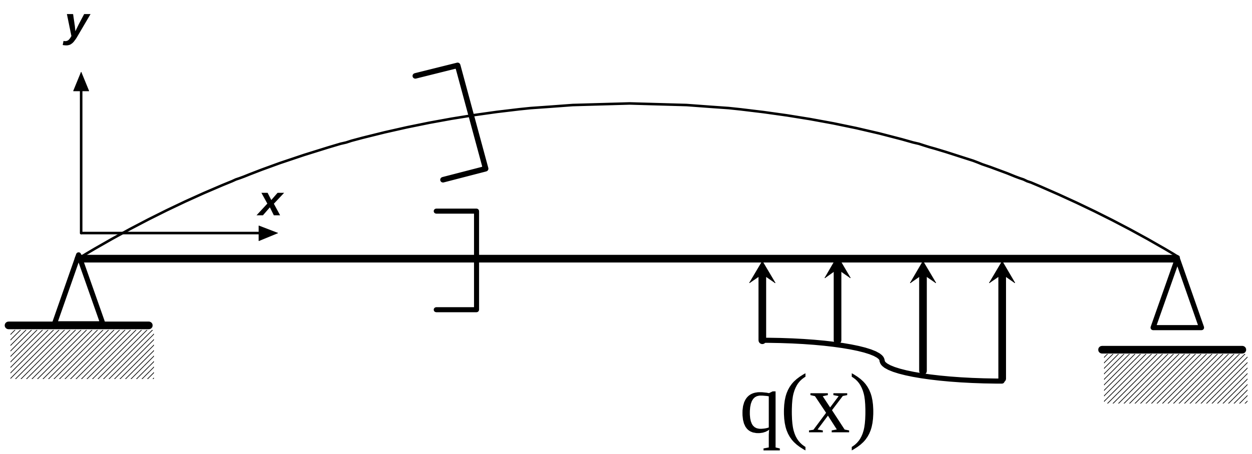
DESPLAZAMIENTOS (Y GIROS) EN FLEXIÓN



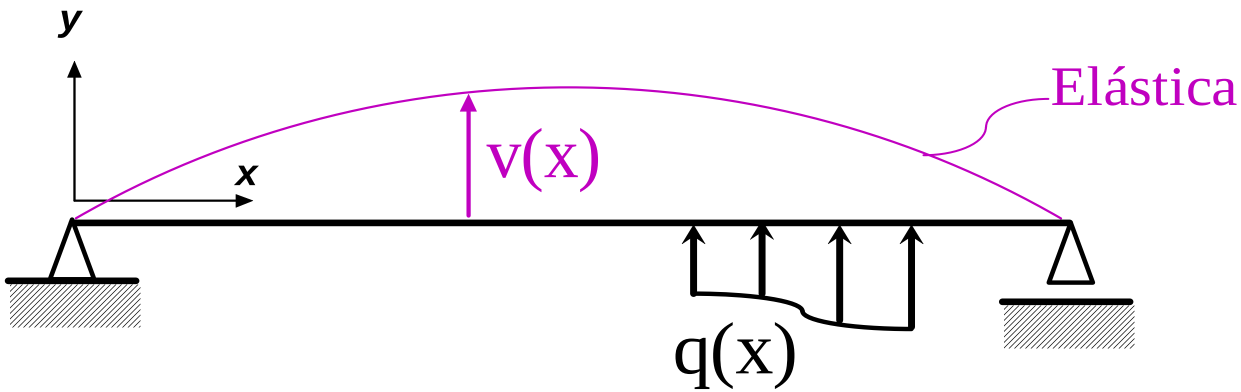
v positivo



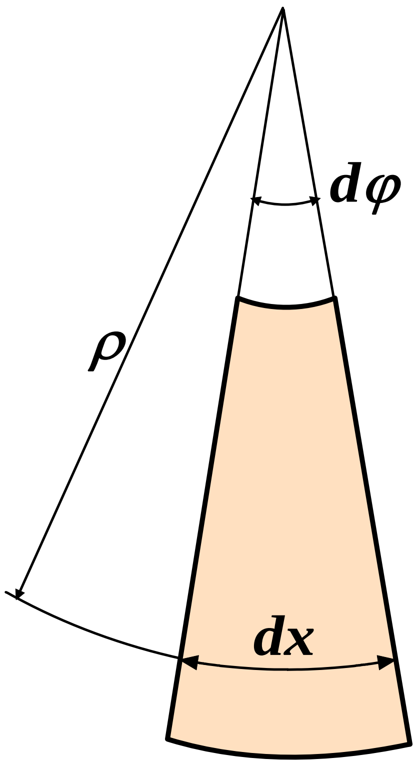
φ positivo

Pequeña deformación:

Ecuación diferencial de la elástica



$$\frac{1}{\rho} = \frac{M_z}{EI_z}$$



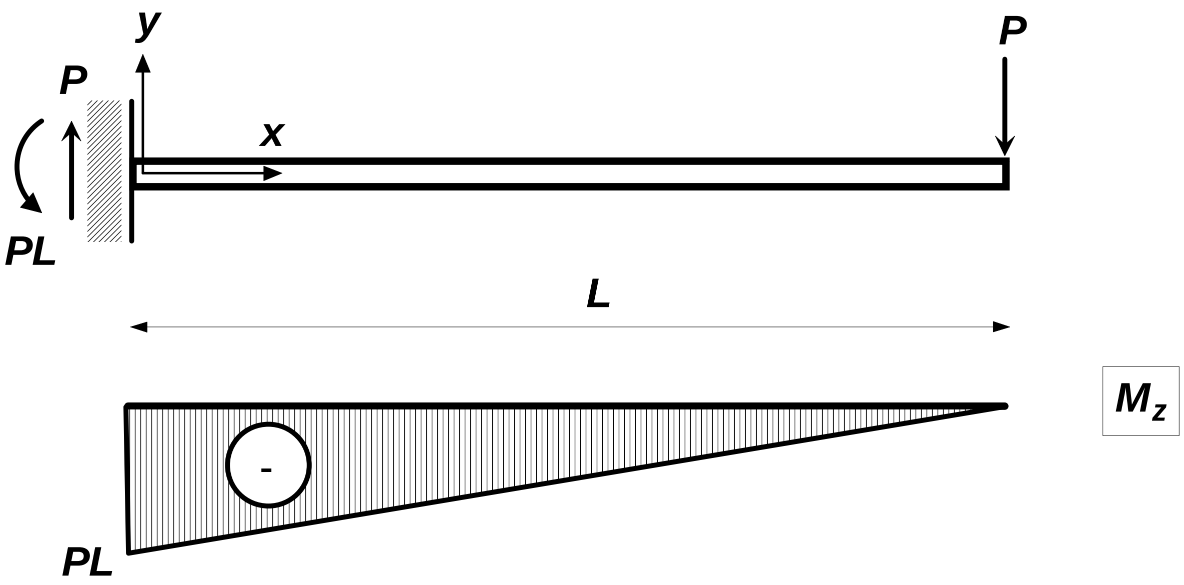
$$\rightarrow \quad \frac{1}{\rho} = \varphi'(x)$$

Pequeña deformación:

$$\varphi(x) \cong v'(x)$$

$$\boxed{v'' = \frac{M_z}{EI_z}} \qquad v^{IV)} = \frac{q_y}{EI_z}$$

Ejemplos



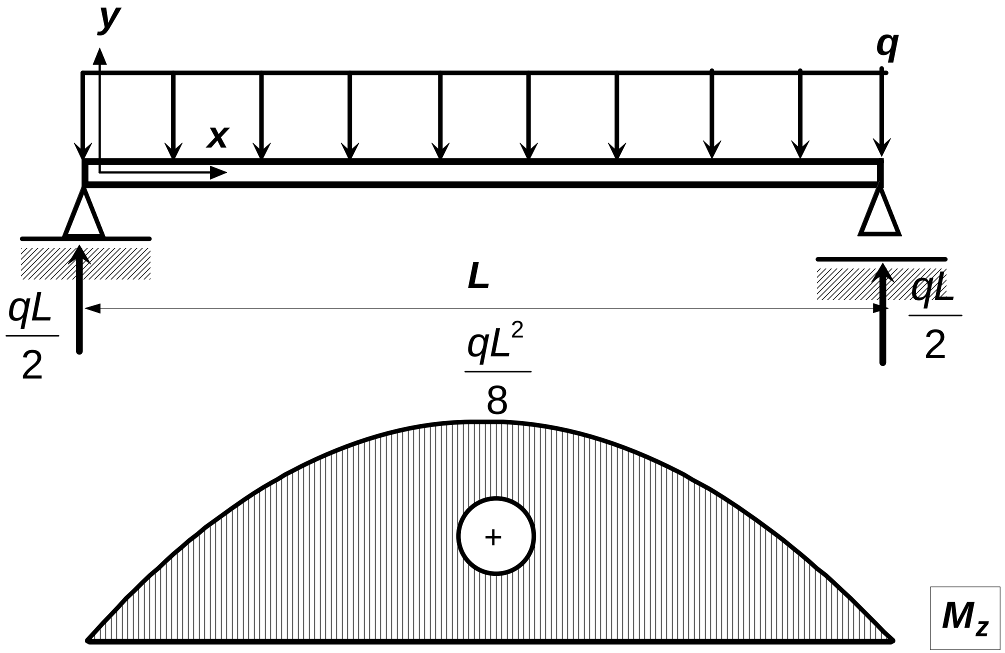
$$M_z = -PL + Px$$

$$EI_z v'' = -PL + Px$$

$$EI_z v(x) = -PL\frac{x^2}{2} + P\frac{x^3}{6}$$

Condiciones de contorno:

Ejemplos



$$M_z = \frac{qL}{2}x - q\frac{x^2}{2}$$

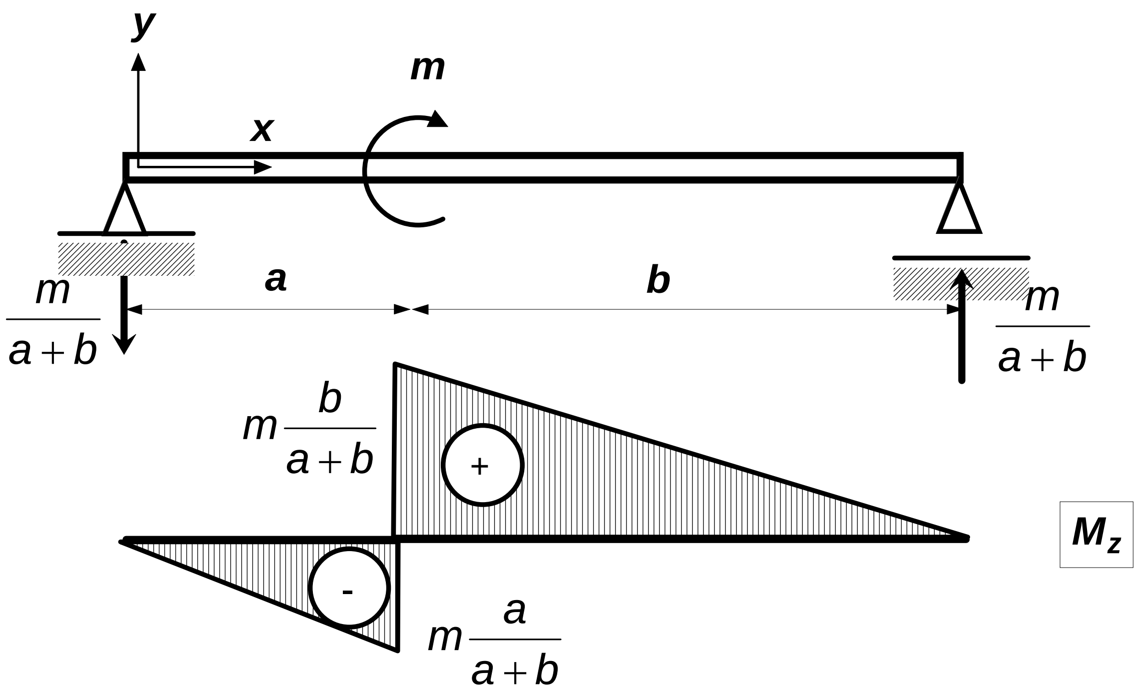
$$EI_z v'' = \frac{qL}{2}x - q\frac{x^2}{2}$$

$$EI_z v(x) = \frac{qL}{2}\frac{x^3}{6} - q\frac{x^4}{24} + C_1 x + C_0$$

Condiciones de contorno: $v(0) = 0 \rightarrow C_0 = 0$

$$v(L) = 0 \rightarrow \quad C_1 = -\frac{qL^4}{24}$$

Ejemplos



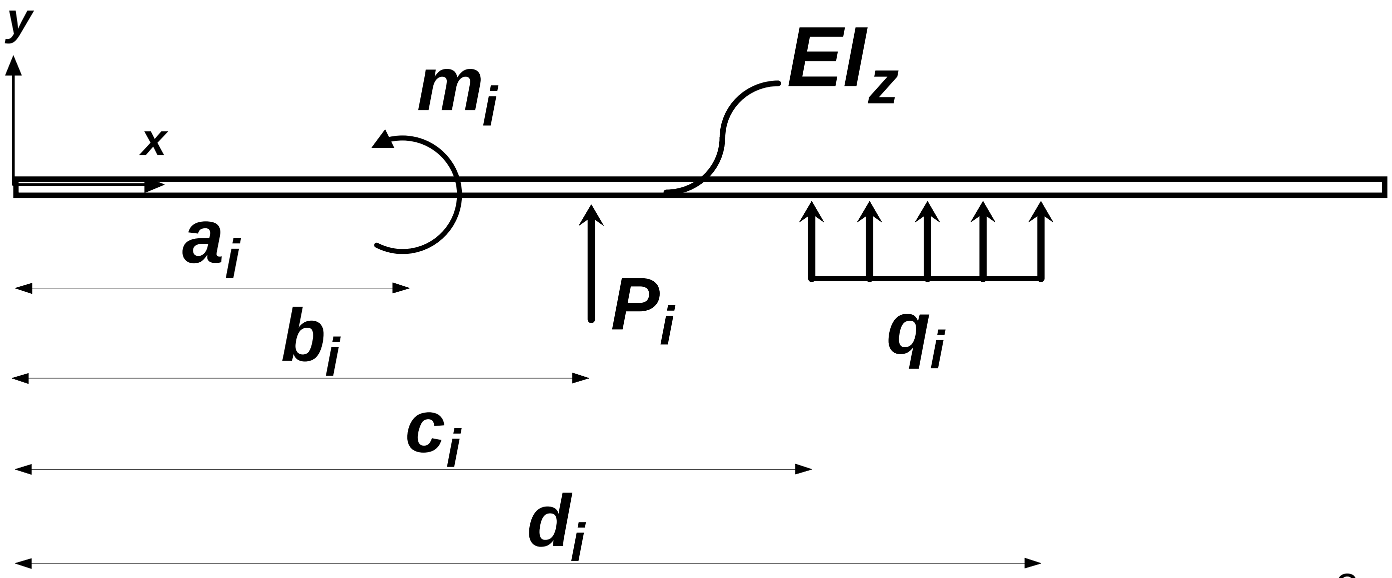
$$\begin{cases} 0 \le x < a & M_z = -\dfrac{m}{a+b}x \\ a \le x & M_z = -\dfrac{m}{a+b}x + m \end{cases}$$

$$\begin{cases} 0 \le x < a & EI_z v'' = -\dfrac{m}{a+b}x \\ a \le x & EI_z v'' = -\dfrac{m}{a+b}x + m \end{cases}$$

$$\begin{cases} 0 \le x < a & EI_z v = -\dfrac{m}{a+b}\dfrac{x^3}{6} \\ a \le x & EI_z v = -\dfrac{m}{a+b}\dfrac{x^3}{6} + m\dfrac{x^2}{2} \end{cases}$$

Condiciones de contorno: $v(0) = 0 \quad v(L) = 0$

Ecuación universal de la elástica

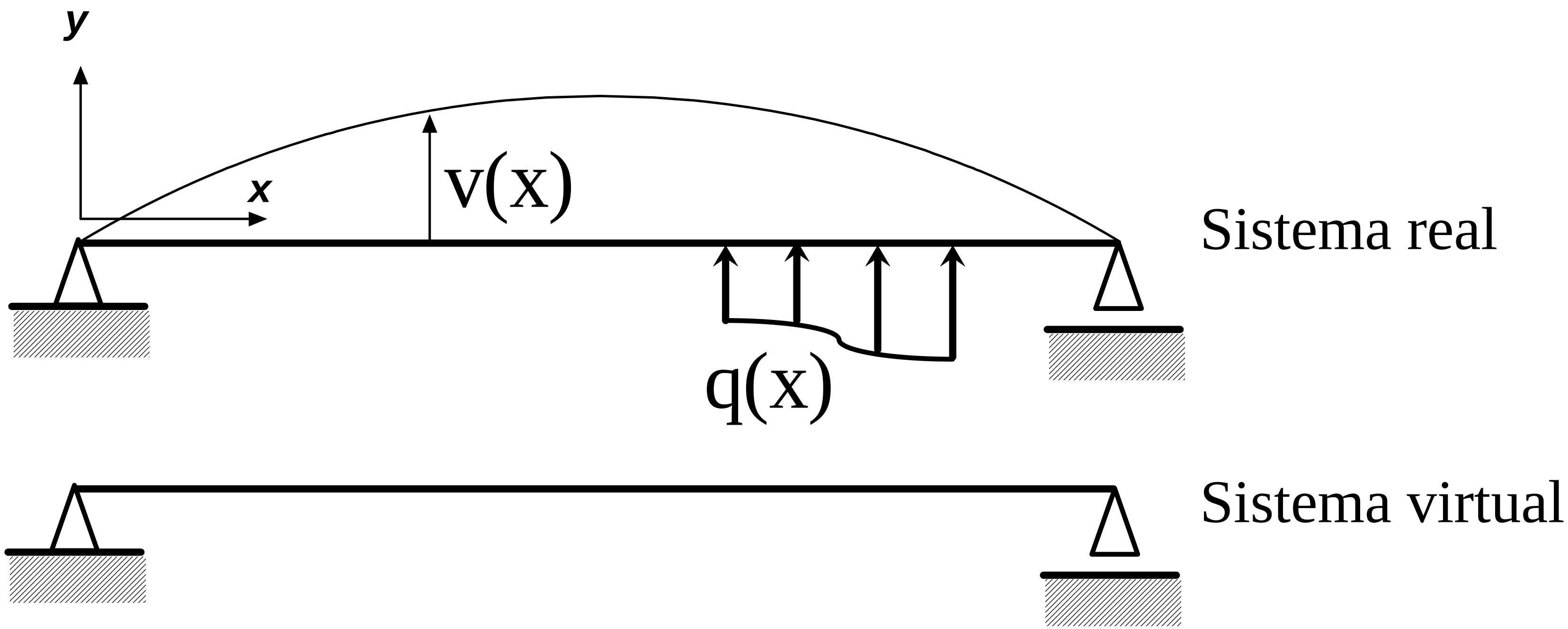


$$EI_z v(x) = \qquad + \qquad - \quad m_i \frac{\langle x - a_i \rangle^2}{2!} +$$

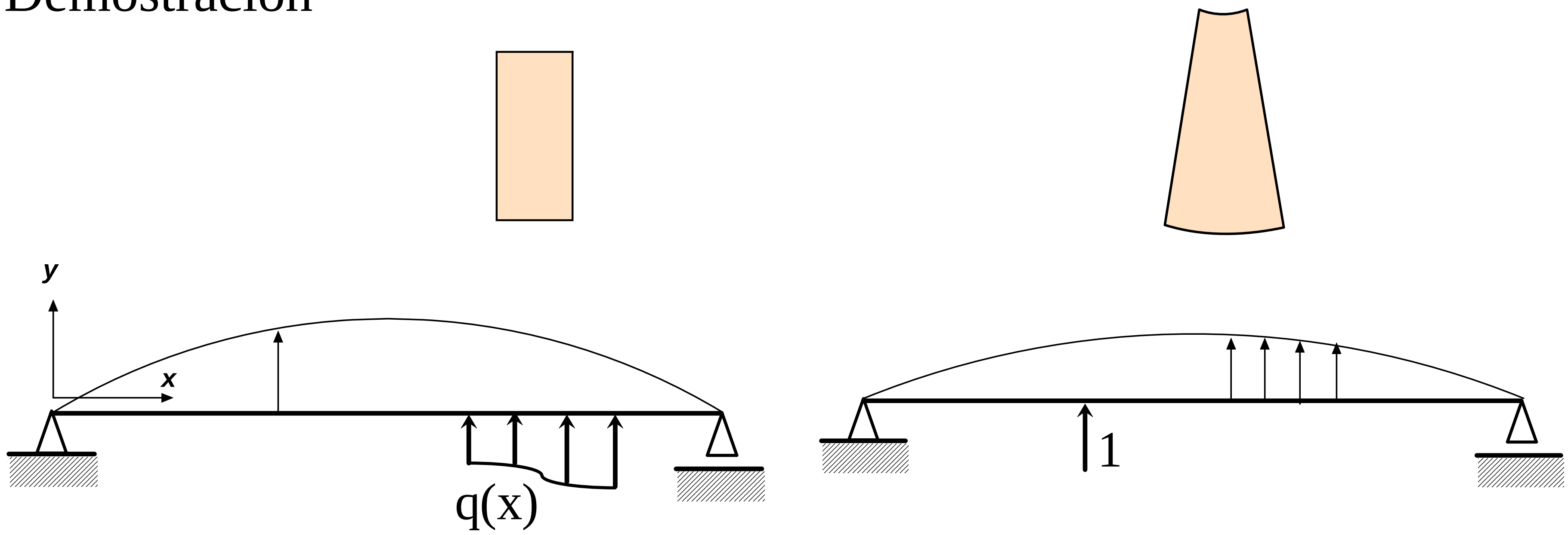
$$+ \quad q_i \frac{\langle x - c_i \rangle^4}{4!}$$

$$\langle x - s \rangle^n = \left\{ \right.$$

Método de la carga unidad



Demostración

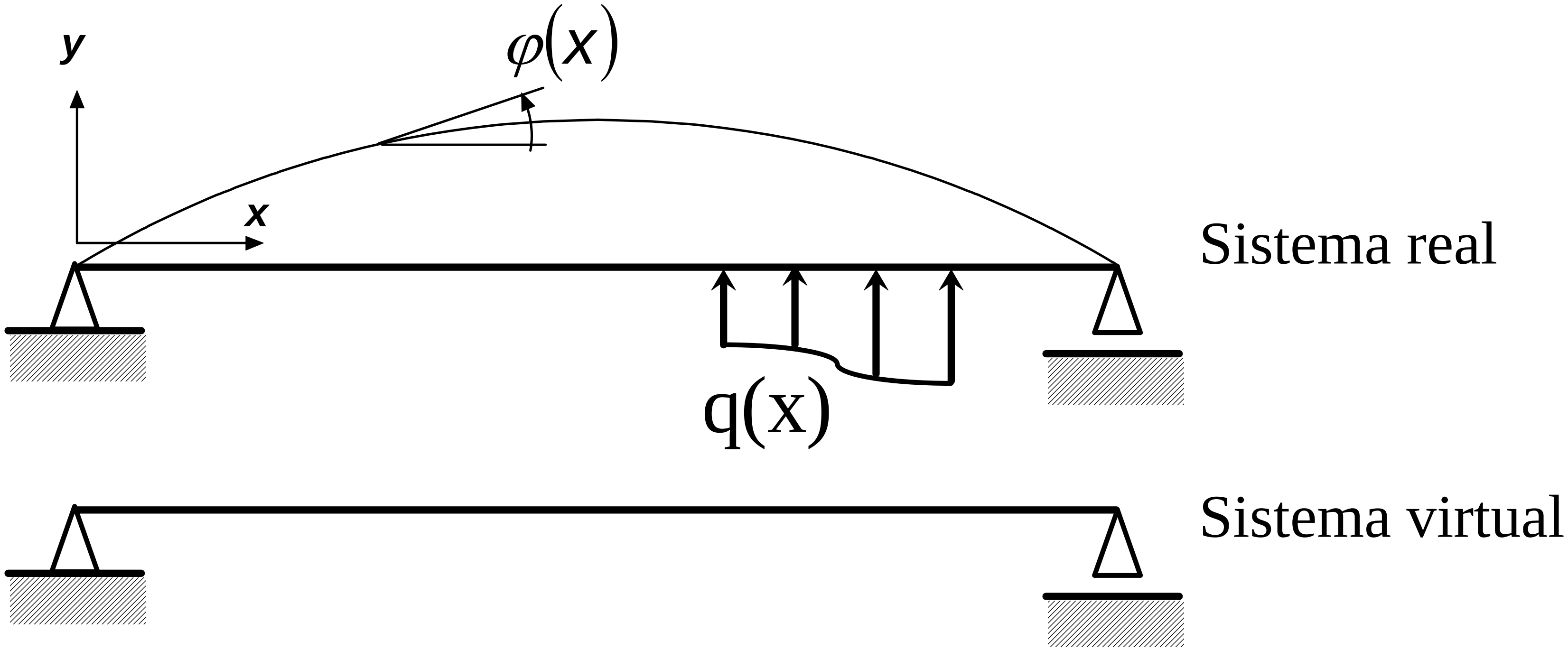


Maxwell-Betti: $$1 \cdot v(x) = \int_0^L q(x) \cdot v_1(x)\,dx$$

Ppio. Conserv energía: $$\int_0^L q(x) \cdot v_1(x)\,dx = \int_0^L M_z(x) \cdot d\varphi_1(x)$$

$$\varphi_1(x) \equiv v_1'(x) \rightarrow \qquad = \frac{M_{z1}}{EI_z} \quad \rightarrow \quad v(x) = \int_0^L \frac{M_z \cdot M_{z1}}{EI_z}\,dx$$

Método del par unidad



$$v'(x) = \int_0^L \frac{M_z \cdot M_{z1}}{EI_z} dx$$